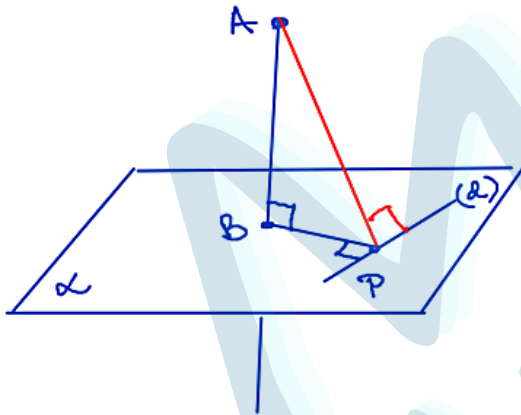


13.03.2021

Recapitulare Teorema celor 3 perpendiculate, reciproce, unghi diedru

T. celor 3 perp :



$$\left. \begin{array}{l} AB \perp \alpha \\ d \subset \alpha \\ BP \perp d \end{array} \right\} \Rightarrow AP \perp d$$

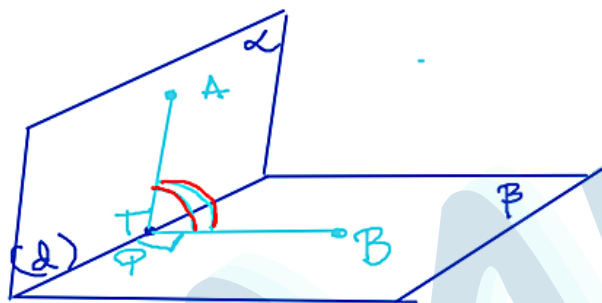
$$\underline{\underline{R1:}} \left. \begin{array}{l} AB \perp d \\ d \subset \alpha \\ AP \perp d \end{array} \right\} \Rightarrow BP \perp d$$

$$\underline{\underline{R2:}} \left. \begin{array}{l} d \subset \alpha \\ BP \subset \alpha \\ BP \perp d \\ A \notin \alpha \\ AP \perp d \\ AB \perp BP \end{array} \right\} \Rightarrow AB \perp \alpha$$

= pag 1. =

Prezentul material este proprietatea intelectuala a societatii Mind Generation SRL si intra sub incidenta drepturilor de autor. Copierea, transmiterea sau comercializarea acestui material sunt strict interzise si se pedepsesc conform legii. Utilizarea lui se poate face doar cu acordul societatii Mind Generation.

Diidru = \sphericalangle între 2 plane



α, β plane

$$d = \alpha \cap \beta$$

$$\forall P \in d$$

Ridicăm din P perpendiculare în cele
2 plane

$$AP \perp d ; BP \perp d$$

$$AP \subset \alpha ; BP \subset \beta$$

$$\left. \begin{array}{l} AP \perp d ; BP \perp d \\ AP \subset \alpha ; BP \subset \beta \end{array} \right\} \Rightarrow (\alpha, \beta) = \widehat{APB}$$

~ ~ ~

① planul $\mathcal{C}(O, R)$, $R = 8\text{cm}$
 Se ridică perpendiculara $OM = 3\text{cm}$.
 Fie $A, B \in \mathcal{C}(O, R)$, $m(\widehat{AB}) = 120^\circ$

 $d(M, AB) = ?$

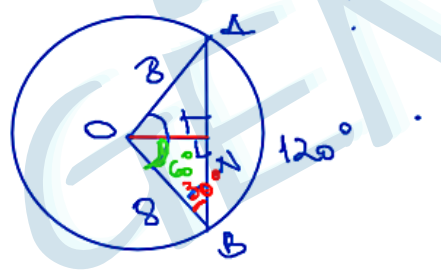
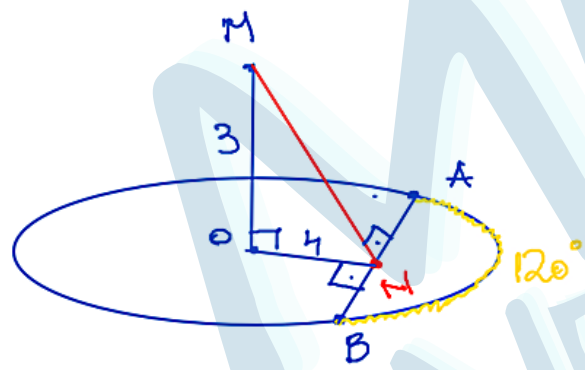
$$\Delta MON \begin{cases} \widehat{MON} = 90^\circ & \text{T.P.} \\ MO = 3\text{cm} \\ ON = 4\text{cm} \end{cases} \Rightarrow MN^2 = MO^2 + ON^2$$

$$\Rightarrow MN^2 = 3^2 + 4^2 = 9 + 16 = 25 \Rightarrow$$

$$\Rightarrow MN = \sqrt{25} = 5\text{cm} \Rightarrow d(M, AB) = 5\text{cm}$$

D: Duceam $MN \perp AB \Rightarrow d(M, AB) = MN$

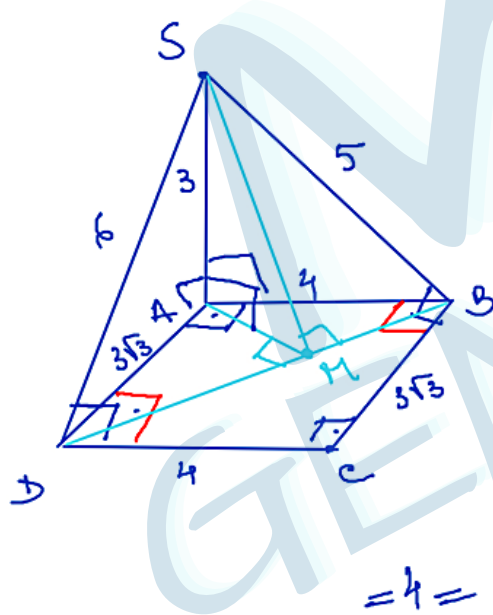
R.T. 3P:
 $OM \perp (\mathcal{C}(O, R))$
 $MN \perp AB$
 $AB \subset (\mathcal{C}(O, R))$ } R.T. 3P. $\Rightarrow ON \perp AB$



ΔOAB isoscel : $OA = OB = R = 8\text{cm}$
 $ON \perp AB$ } $\Rightarrow ON$ mediană, bisectoare } \Rightarrow

$\widehat{AOB} = 120^\circ \Rightarrow \widehat{BOB} = 120^\circ$
 $\Rightarrow \widehat{BON} = 60^\circ \Rightarrow \widehat{OBN} = 30^\circ$ T. 30°
 $\sim 3 \sim \Rightarrow ON = \frac{OB}{2} = \frac{8}{2} = 4\text{cm}$

② ABCD patrulater
 $\widehat{DAB} = 90^\circ$
 $SA \perp (BCD)$
 $SA = 3 \text{ cm}$
 $d(S, BC) = SB = 5 \text{ cm}$
 $d(S, DC) = SD = 6 \text{ cm}$



C : a) $AB = ?$
 b) $AD = ?$
 c) $P_{ABCD} = ?$
 d) $d(S, BD) = ?$

D : $d(S, BC) = SB \Rightarrow SB \perp BC$ } R.3.L
 $SA \perp (ABC)$ } $\Rightarrow AB \perp BC$
 $BC \subset (ABC)$

$d(S, DC) = SD \Rightarrow SD \perp DC$ } R.3.L
 $SA \perp (ABC)$ } $\Rightarrow AD \perp DC$
 $DC \subset (ABC)$

$\Rightarrow ABCD = \text{dreptunghi}$!
 $\triangle SAB \begin{cases} SA = 3 \text{ cm} \\ SB = 5 \text{ cm} \\ \widehat{SAB} = 90^\circ \end{cases} \xrightarrow{\text{T.P.}} SB^2 = SA^2 + AB^2 \Rightarrow AB^2 = 5^2 - 3^2 = 25 - 9$
 $\Rightarrow AB = 4 \text{ cm} \Rightarrow DC = 4 \text{ cm}$

$$b) \triangle SAD \begin{cases} SA = 3 \text{ cm} \\ SD = 6 \text{ cm} \\ \hat{SAD} = 90^\circ \end{cases} \xrightarrow{\text{T.P.}} SD^2 = SA^2 + AD^2 \Rightarrow AD^2 = 6^2 - 3^2 = 36 - 9 = 27 \Rightarrow AD = \sqrt{27} = 3\sqrt{3} \text{ cm} \Rightarrow BC = 3\sqrt{3} \text{ cm}$$

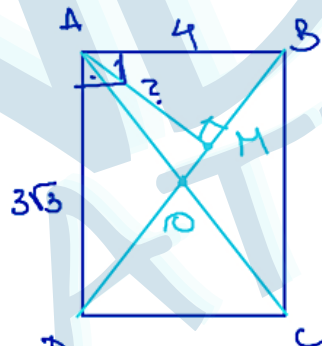
$$c) P_{ABCD} = 2(AB + AD) = 2(5 + 3\sqrt{3}) \text{ cm}$$

$$d) \text{Ducem } SM \perp BD \left. \begin{array}{l} SA \perp (ABC) \\ BD \subset (ABC) \end{array} \right\} \text{R. 31} \Rightarrow AM \perp BD$$

$$\triangle ABD \begin{cases} \hat{A} = 90^\circ \\ AB = 4 \text{ cm} \\ AD = 3\sqrt{3} \text{ cm} \end{cases} \xrightarrow{\text{T.P.}} BD^2 = AB^2 + AD^2$$

$$BD^2 = 16 + 27 = 43$$

$$\Rightarrow BD = \sqrt{43} \text{ cm}$$



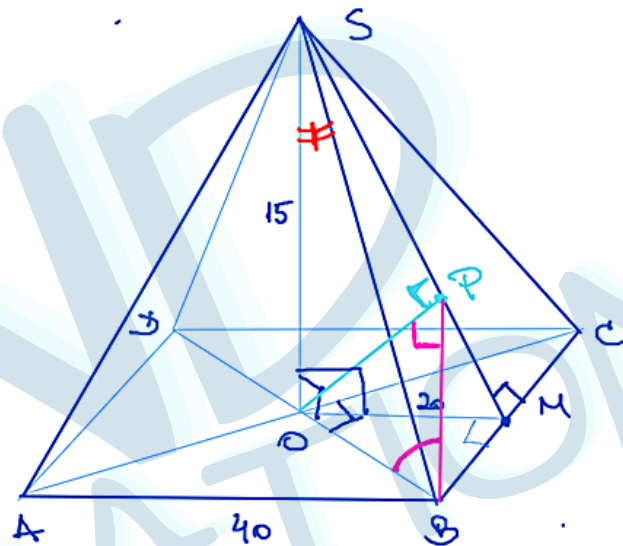
$$AM = \frac{AB \cdot AD}{BD} = \frac{4 \cdot 3\sqrt{3}}{\sqrt{43}} = \frac{12\sqrt{129}}{43} \text{ cm}$$

$$\triangle SAM \begin{cases} \hat{SAM} = 90^\circ \\ SA = 3 \text{ cm} \\ AM = \frac{12\sqrt{129}}{43} \end{cases} \xrightarrow{\text{T.P.}} SM^2 = SA^2 + AM^2 = 9 + \frac{12^2 \cdot 129}{43^2} = \frac{3^2(43^2 + 4^2 \cdot 129)}{43^2}$$

$$\Rightarrow SM = \frac{3}{43} \sqrt{43^2 + 4^2 \cdot 129} = \dots = \frac{3\sqrt{91}}{\sqrt{43}} = \frac{3\sqrt{\dots}}{43}$$

③ SABCD pir. patr. regulată
 $AB = 40 \text{ cm}$
 $SO = 15 \text{ cm}$

- a) $\text{tg}(\widehat{SB, (SAC)}) = ?$ ✓
 b) $d(O, (SBC)) = ?$ ✓
 c) $\sin(\widehat{OB, (SBC)}) = ?$ ✓
 d) $(AD, SM), M = \text{mijl. } |BC|$ ✓



$$\stackrel{D}{=} a) \widehat{SB, (SAC)} = \widehat{SB, \text{pr}_{(SAC)} SB}$$

pr_(SAC) SB = SO deoarece: $BO \perp (SAC) \left(\begin{array}{l} BO \perp AC \\ BO \perp SO \end{array} \Rightarrow BO \perp (SAC) \right) \Rightarrow$

$$\Rightarrow \widehat{SB, (SAC)} = \widehat{SB, SO} = \widehat{BSO}$$

$$\Delta SOB \begin{cases} \widehat{SOB} = 90^\circ \\ SO = 15 \text{ cm} \\ BO = \frac{BD}{2} = \frac{AB\sqrt{2}}{2} = \frac{40\sqrt{2}}{2} \end{cases}$$

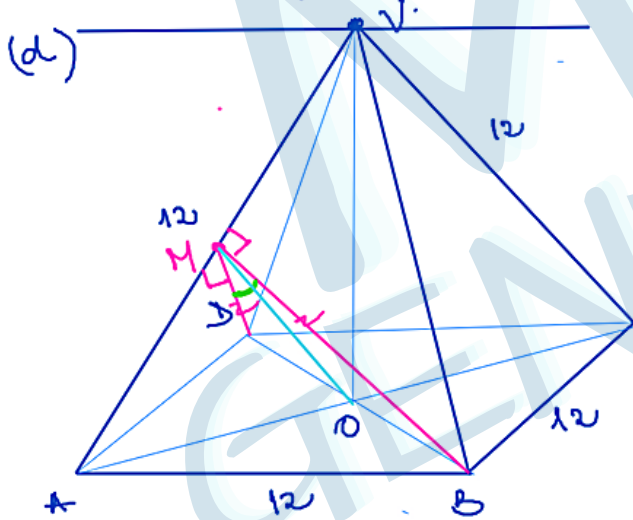
$$= b = \Rightarrow SO = 20\sqrt{2} \text{ cm}$$

$$\Rightarrow \text{tg} \widehat{BSO} = \frac{OB}{SO} = \frac{20\sqrt{2}}{15 \cdot 3} = \frac{4\sqrt{2}}{3}$$

$$\Rightarrow \sin(\widehat{OBP}) = \frac{\frac{\sqrt{2}}{2} \cdot 12^3}{25\sqrt{2}} = \frac{3\sqrt{2}}{10} \quad \left(\text{Vf: } \frac{3\sqrt{2}}{10} < 1 \checkmark \right)$$

$$d) \left. \begin{array}{l} (\widehat{AD, SM}) = (\widehat{BC, SM}) \text{ deoarece } AD \parallel BC \\ SM \perp BC \text{ (pt. b.)} \end{array} \right\} \Rightarrow (\widehat{AD, SM}) = 90^\circ$$

④ VABCD pir. patr. regulată; VAB echilaterale: VA = VB = VC = VD = AB = 12 cm



a) $\sin(\widehat{(VAB), (VAD)}) = ?$ ✓

b) $\sin(\widehat{(VAB), (VCD)}) = ?$

a) $(VAB) \cap (VAD) = VA$

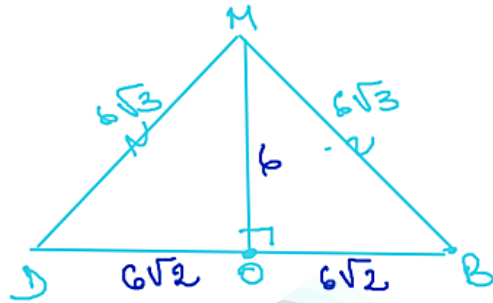
$$\left. \begin{array}{l} \text{Ducem } BM \perp VA \\ \Delta VAB \text{ ech.} \end{array} \right\} \Rightarrow M = \text{mijl } |VA| \left. \begin{array}{l} \Delta VAD \text{ ech.} \end{array} \right\} \Rightarrow DM \perp VA$$

$$\Rightarrow \widehat{((VAB), (VAD))} = \widehat{BMD}$$

$$\sin \widehat{MJB} = \frac{MD \cdot MB \cdot \sin \widehat{BMD}}{2} \quad (1)$$

$$\boxed{= 8 =}$$

$$\triangle VAB \equiv \triangle VAD \text{ echilat} \Rightarrow \underline{BM} = \underline{DM} = \frac{12\sqrt{3}}{2} = \underline{6\sqrt{3} \text{ cm}}$$



O = mijl. DBI } $\Rightarrow MO \perp DB$
 M DB isoscel

ABCD pătrat } $\Rightarrow BD = \underline{12\sqrt{2} \text{ cm}} \Rightarrow DO = 6\sqrt{2} \text{ cm}$
 AB = 12 cm

$$\triangle MDO \left\{ \begin{array}{l} \text{T.P} \\ \Rightarrow MD^2 = MO^2 + DO^2 \end{array} \right. \Rightarrow MO^2 = MD^2 - DO^2 = (6\sqrt{3})^2 - (6\sqrt{2})^2 = 6^2 \cdot 3 - 6^2 \cdot 2 = 6^2(3-2) = 6^2$$

$$\Rightarrow MO = \sqrt{6^2} = \underline{6 \text{ cm}} \Rightarrow$$

$$\Rightarrow A_{MDB} = \frac{MO \cdot DB}{2} = \frac{6 \cdot 12\sqrt{2}}{2} = 36\sqrt{2} \text{ cm}^2 \quad (2)$$

$$\text{Din (1) \& (2)} \Rightarrow \frac{6\sqrt{3} \cdot 6\sqrt{3} \cdot \sin(\widehat{BMD})}{2} = 36\sqrt{2} \Leftrightarrow 54 \sin(\widehat{BMD}) = 36\sqrt{2} \Leftrightarrow$$

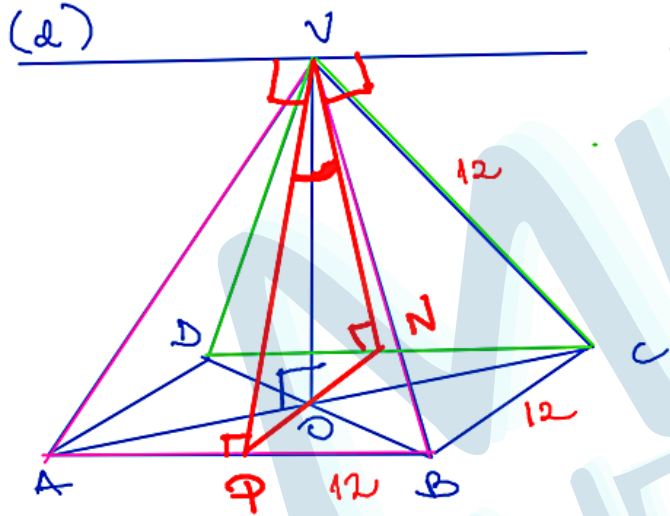
$$\Leftrightarrow \sin(\widehat{BMD}) = \frac{36\sqrt{2}}{54} = \frac{2\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

$$\frac{2\sqrt{2}}{3} < 1$$

$$\frac{2\sqrt{2}}{3} = \frac{\sqrt{8}}{3} \Rightarrow \frac{2\sqrt{2}}{3} < 1 \checkmark$$

e) $(VAB) \cap (VDC) = d$, $d \parallel AB \parallel DC$
 $d \parallel (ABC)$

Deci $VP \perp AB \Rightarrow VP \perp d$
 $VN \perp DC \Rightarrow VN \perp d \Rightarrow$
 $\Rightarrow (\widehat{VAB}, \widehat{VDC}) = \widehat{PVN}$



$VP = VN =$ înălțimi în Δ ech. de latură 12 cm \Rightarrow
 $\Rightarrow VP = VN = \frac{12\sqrt{3}}{2} = 6\sqrt{3}$ cm

$PN = 12$ cm

$A_{VPN} = \frac{VP \cdot VN \cdot \sin(\widehat{PVN})}{2} = \frac{VO \cdot PN}{2} (*)$

$\Delta VAO \begin{cases} \widehat{VOA} = 90^\circ \\ VA = 12 \text{ cm} \\ AO = \frac{12\sqrt{2}}{2} = 6\sqrt{2} \text{ cm} \end{cases}$

T.P. $\Rightarrow VA^2 = VO^2 + AO^2 \Rightarrow$

$\Rightarrow VO^2 = 12^2 - (6\sqrt{2})^2 = 144 - 72 = 72 \Rightarrow VO = \sqrt{72} = 6\sqrt{2}$ cm

$(*) \Rightarrow \frac{6\sqrt{3} \cdot 6\sqrt{3} \cdot \sin(\widehat{PVN})}{2} = \frac{6\sqrt{2} \cdot 12 \cdot 2}{2} \Rightarrow \sin(\widehat{PVN}) = \frac{2\sqrt{2}}{3}$
 $\boxed{= 10 =}$

Tema:

In paralelogramul ABCD cu $AB=6\text{cm}$, $BC=12\text{cm}$ si AC perpendicular pe AB , se ridica in pct. O de intersectie a diagonalelor perpendiculara pe planul sau, $MO=3\text{cm}$. Calculati:

- Masura unghiului plan corespunzator diedrului format de (MAB) si (ABC)
- Tangenta unghiului plan corespunzator diedrului format de (MBC) si (ABC)

= 11 =