

Mind Generation
Centru de Matematica si Informatica

Ecuatia $x^2 = a$
Algebra - cls a VII-a

Notiuni teoretice

1. Formula de calcul prescurtat: $(a-b)(a+b) = a^2 - b^2$

2. Fie ecuatia: $x^2 = a$

Cazuri: 1. $a < 0 \Rightarrow x^2 < 0$ nu se poate (un pătrat nu poate fi negativ) $\Rightarrow x \in \emptyset$

2. $a = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

3. $a > 0 \Rightarrow$ rezolvăm: $x^2 = a \Leftrightarrow x^2 - a = 0 \Leftrightarrow x^2 - (\sqrt{a})^2 = 0 \Leftrightarrow$
($a > 0$)

$$\Leftrightarrow (x - \sqrt{a})(x + \sqrt{a}) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x - \sqrt{a} = 0 \Leftrightarrow x = \sqrt{a} \\ \text{sau} \\ x + \sqrt{a} = 0 \Leftrightarrow x = -\sqrt{a} \end{cases} \Rightarrow x \in \{-\sqrt{a}, \sqrt{a}\}$$

Exemple rezolvate:

Să se rezolve în \mathbb{R} ecuațiile:

1. $x^2 = \frac{16}{25}$ $\Leftrightarrow x^2 - \frac{16}{25} = 0 \Leftrightarrow x^2 - \left(\frac{4}{5}\right)^2 = 0 \Leftrightarrow (x - \frac{4}{5})(x + \frac{4}{5}) = 0 \Leftrightarrow$ $\begin{cases} x - \frac{4}{5} = 0 \Leftrightarrow x = \frac{4}{5} \\ \text{sau} \\ x + \frac{4}{5} = 0 \Leftrightarrow x = -\frac{4}{5} \end{cases} \Leftrightarrow$
 $\Leftrightarrow x \in \left\{ -\frac{4}{5}; \frac{4}{5} \right\}$

2. $x^2 = 0,0027$ $\Leftrightarrow x^2 = \frac{27}{10000} \Leftrightarrow x^2 - \frac{27}{10000} = 0 \Leftrightarrow x^2 - \left(\sqrt{\frac{27}{10000}}\right)^2 = 0 \Leftrightarrow$
 $\Leftrightarrow x^2 - \left(\frac{3\sqrt{3}}{100}\right)^2 = 0 \Leftrightarrow (x - \frac{3\sqrt{3}}{100})(x + \frac{3\sqrt{3}}{100}) = 0 \Leftrightarrow$
 $\Leftrightarrow \begin{cases} x - \frac{3\sqrt{3}}{100} = 0 \Leftrightarrow x = \frac{3\sqrt{3}}{100} \\ \text{sau} \\ x + \frac{3\sqrt{3}}{100} = 0 \Leftrightarrow x = -\frac{3\sqrt{3}}{100} \end{cases} \Leftrightarrow x \in \left\{ -\frac{3\sqrt{3}}{100}; \frac{3\sqrt{3}}{100} \right\}$

3. $x^2 + 1 = 10$ $\Leftrightarrow x^2 = 10 - 1 \Leftrightarrow x^2 = 9 \Leftrightarrow x^2 - 9 = 0 \Leftrightarrow x^2 - 3^2 = 0 \Leftrightarrow (x - 3)(x + 3) = 0 \Leftrightarrow$
 $\Leftrightarrow \begin{cases} x - 3 = 0 \Rightarrow x = 3 \\ \text{sau} \\ x + 3 = 0 \Rightarrow x = -3 \end{cases} \Leftrightarrow x \in \{-3, 3\}$

4. $\underline{2x^2 + 5 = 4} \Leftrightarrow 2x^2 = 4 - 5 \Leftrightarrow 2x^2 = -1 \Leftrightarrow x^2 = -\frac{1}{2} < 0 \Rightarrow x \in \emptyset$

5. $\underline{10x^2 + 2 = 2} \Leftrightarrow 10x^2 = 2 - 2 \Leftrightarrow 10x^2 = 0 \Rightarrow x = 0 \Rightarrow x \in \{0\}$

6. $\underline{x^2 - \sqrt{3} = -2} \Leftrightarrow x^2 = \sqrt{3} - 2$

trebuie să det. dacă $\sqrt{3} - 2 > 0$ sau $\sqrt{3} - 2 < 0$

$\sqrt{3} - 2 = \sqrt{3} - \sqrt{4} < 0 \Rightarrow \sqrt{3} - 2 < 0 \Rightarrow x^2 - \sqrt{3} - 2 < 0 \Rightarrow x \in \emptyset$

7. $\underline{-4x^2 - 3 = 6} \Leftrightarrow -4x^2 = 6 + 3 \Leftrightarrow -4x^2 = 9 \mid : -4 \Leftrightarrow x^2 = -\frac{9}{4} < 0 \Rightarrow x \in \emptyset$

8. $\underline{\frac{1}{4}x^2 + 3 = \frac{1}{4}} \Leftrightarrow \frac{1}{4}x^2 = \frac{1}{4} - 3 \Leftrightarrow \frac{1}{4}x^2 = \frac{1-12}{4} \mid \cdot 4 \Leftrightarrow x^2 = -11 < 0 \Rightarrow x \in \emptyset$

9. $\underline{\frac{x}{3} = \frac{3}{x}} \Leftrightarrow x \cdot x = 3 \cdot 3 \Leftrightarrow x^2 = 3^2 \Leftrightarrow x^2 - 3^2 = 0 \Leftrightarrow (x-3)(x+3) = 0 \Leftrightarrow \begin{cases} x-3=0 \Rightarrow x=3 \neq 0 \\ \text{sau} \\ x+3=0 \Rightarrow x=-3 \neq 0 \end{cases}$
(produsul mezilor =
= produsul extremilor) $\Rightarrow x \in \{-3, 3\}$

10. $\underline{\frac{3}{x-1} = \frac{4(x-1)}{27}} \Leftrightarrow 3 \cdot 27 = 4 \cdot (x-1)^2 \Leftrightarrow 81 = 4(x-1)^2 \Leftrightarrow (x-1)^2 = \frac{81}{4} \Leftrightarrow (x-1)^2 - \left(\frac{9}{2}\right)^2 = 0 \Leftrightarrow$

$$\Leftrightarrow \left[(x-1) - \frac{9}{2} \right] \cdot \left[(x-1) + \frac{9}{2} \right] = 0 \Leftrightarrow \left(x-1 - \frac{9}{2} \right) \left(x-1 + \frac{9}{2} \right) = 0 \Leftrightarrow \begin{cases} \nearrow x-1 - \frac{9}{2} = 0 \Rightarrow x = 1 + \frac{9}{2} \Rightarrow x = \frac{11}{2} \\ \text{sau} \\ \searrow x-1 + \frac{9}{2} = 0 \Rightarrow x = 1 - \frac{9}{2} \Rightarrow x = -\frac{7}{2} \end{cases}$$

$$\Rightarrow x \in \left\{ -\frac{7}{2}; \frac{11}{2} \right\}$$

$$11. \underline{(4x-1)^2 = \frac{1}{4}} \Leftrightarrow (4x-1)^2 - \left(\frac{1}{2}\right)^2 = 0 \Leftrightarrow \left[(4x-1) - \frac{1}{2} \right] \left[(4x-1) + \frac{1}{2} \right] = 0 \Leftrightarrow (4x-1 - \frac{1}{2})(4x-1 + \frac{1}{2}) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \nearrow 4x-1 - \frac{1}{2} = 0 \Rightarrow 4x = 1 + \frac{1}{2} \Rightarrow 4x = \frac{3}{2} \quad | :4 \Rightarrow x = \frac{3}{8} \\ \text{sau} \\ \searrow 4x-1 + \frac{1}{2} = 0 \Rightarrow 4x = 1 - \frac{1}{2} \Rightarrow 4x = \frac{1}{2} \quad | :4 \Rightarrow x = \frac{1}{8} \end{cases} \Rightarrow x \in \left\{ \frac{1}{8}; \frac{3}{8} \right\}$$

$$12. \underline{(x+\sqrt{2})^2 = 8} \Leftrightarrow (x+\sqrt{2})^2 - 8 = 0 \Leftrightarrow (x+\sqrt{2})^2 - (\sqrt{8})^2 = 0 \Leftrightarrow (x+\sqrt{2})^2 - (2\sqrt{2})^2 = 0 \Leftrightarrow$$

$$\sqrt{8} = 2\sqrt{2}$$

$$\Leftrightarrow \left[(x+\sqrt{2}) - 2\sqrt{2} \right] \cdot \left[(x+\sqrt{2}) + 2\sqrt{2} \right] = 0 \Leftrightarrow (x+\sqrt{2} - 2\sqrt{2})(x+\sqrt{2} + 2\sqrt{2}) = 0 \Leftrightarrow$$

$$\Leftrightarrow (x - \sqrt{2})(x + 3\sqrt{2}) = 0 \Leftrightarrow \begin{cases} \nearrow x - \sqrt{2} = 0 \Rightarrow x = \sqrt{2} \\ \text{sau} \\ \searrow x + 3\sqrt{2} = 0 \Rightarrow x = -3\sqrt{2} \end{cases} \Rightarrow x \in \left\{ -3\sqrt{2}; \sqrt{2} \right\}$$

$$13. \frac{3}{2x-1} = \frac{2x-1}{75} \Leftrightarrow 3 \cdot 75 = (2x-1)(2x-1) \Leftrightarrow 3 \cdot 3 \cdot 25 = (2x-1)^2 \Leftrightarrow 3^2 \cdot 5^2 - (2x-1)^2 = 0 \Leftrightarrow$$

$$\frac{3}{2x-1} \neq 0 \Leftrightarrow x \neq \frac{1}{2} \quad \text{Pag.4} \quad \Leftrightarrow (3 \cdot 5)^2 - (2x-1)^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 15^2 - (2x-1)^2 = 0 \Leftrightarrow [15 - (2x-1)] \cdot [15 + (2x-1)] = 0 \Leftrightarrow (15 - 2x + 1)(15 + 2x - 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow (16 - 2x)(14 + 2x) = 0 \Leftrightarrow \begin{cases} 16 - 2x = 0 \Leftrightarrow 2x = 16 \quad | :2 \Leftrightarrow x = 8 \\ \text{sau} \\ 14 + 2x = 0 \Leftrightarrow 2x = -14 \quad | :2 \Leftrightarrow x = -7 \end{cases} \Leftrightarrow x \in \underline{\underline{\{-7; 8\}}}$$

$$14. \underline{x^2 = 151(6)} \Leftrightarrow x^2 = \frac{116-11}{90} \Leftrightarrow x^2 = \frac{105}{90} \stackrel{12}{\cdot 6} \stackrel{7}{\cdot 7} \Leftrightarrow x^2 = \frac{7}{6} \Leftrightarrow x^2 - \left(\sqrt{\frac{7}{6}}\right)^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow \left(x - \sqrt{\frac{7}{6}}\right)\left(x + \sqrt{\frac{7}{6}}\right) = 0 \Leftrightarrow \begin{cases} x - \sqrt{\frac{7}{6}} = 0 \Leftrightarrow x = \sqrt{\frac{7}{6}} = \frac{\sqrt{7}}{\sqrt{6}} = \frac{\sqrt{42}}{6} \\ \text{sau} \\ x + \sqrt{\frac{7}{6}} = 0 \Leftrightarrow x = -\sqrt{\frac{7}{6}} = -\frac{\sqrt{7}}{\sqrt{6}} = -\frac{\sqrt{42}}{6} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow x \in \underline{\underline{\left\{-\frac{\sqrt{42}}{6}; \frac{\sqrt{42}}{6}\right\}}}$$

$$15. \underline{x^2 - \sqrt{3} = \sqrt{(\sqrt{3}-2)^2}} \Leftrightarrow x^2 - \sqrt{3} = |\sqrt{3}-2| \Leftrightarrow x^2 - \sqrt{3} = 2 - \sqrt{3} \Leftrightarrow x^2 = 2 - \sqrt{3} + \sqrt{3} \Leftrightarrow x^2 = 2 \Leftrightarrow$$

$$\Rightarrow x^2 - 2 = 0 \Leftrightarrow x^2 - (\sqrt{2})^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow (x - \sqrt{2})(x + \sqrt{2}) = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x - \sqrt{2} = 0 \Leftrightarrow x = \sqrt{2} \\ \text{sau} \\ x + \sqrt{2} = 0 \Leftrightarrow x = -\sqrt{2} \end{cases} \Leftrightarrow x \in \underline{\underline{\{-\sqrt{2}; \sqrt{2}\}}}$$

Trebuie să determinăm
 semnul pt. $\sqrt{3}-2$

$$\sqrt{3}-2 = \sqrt{3}-\sqrt{4} < 0 \Rightarrow |\sqrt{3}-2| = -(\sqrt{3}-2) = 2-\sqrt{3}$$

pt. c. $\sqrt{3} < \sqrt{4}$

16. $\underline{x^2 = 2^{50} + 2^{53}}$ $\Leftrightarrow x^2 = 2^{50}(1+2^3) \Leftrightarrow x^2 = 2^{50}(1+8) \Leftrightarrow x^2 = 2^{50} \cdot 9 \Leftrightarrow x^2 = 2^{50} \cdot 3^2 \Leftrightarrow$
 $\Leftrightarrow x^2 - 2^{50} \cdot 3^2 = 0 \Leftrightarrow x^2 - (2^{25} \cdot 3)^2 = 0 \Leftrightarrow (x - 2^{25} \cdot 3)(x + 2^{25} \cdot 3) = 0 \Leftrightarrow$
 $\Leftrightarrow \begin{cases} \rightarrow x - 2^{25} \cdot 3 = 0 \Leftrightarrow x = 3 \cdot 2^{25} \\ \downarrow \text{sau} \\ x + 2^{25} \cdot 3 = 0 \Leftrightarrow x = -3 \cdot 2^{25} \end{cases} \Rightarrow \underline{x \in \{-3 \cdot 2^{25}; 3 \cdot 2^{25}\}}$

17. $\underline{x^2 = 3^{23} - 3^{20} - 3^{20}}$ $\Leftrightarrow x^2 = 3^{20}(3^3 - 1 - 1) \Leftrightarrow x^2 = 3^{20}(27 - 1 - 1) \Leftrightarrow x^2 = 3^{20} \cdot 25 \Leftrightarrow$
 $\Leftrightarrow x^2 = 3^{20} \cdot 5^2 \Leftrightarrow x^2 - 3^{20} \cdot 5^2 = 0 \Leftrightarrow x^2 - (3^{10} \cdot 5)^2 = 0 \Leftrightarrow$
 $\Leftrightarrow (x - 3^{10} \cdot 5)(x + 3^{10} \cdot 5) = 0 \Leftrightarrow \begin{cases} \rightarrow x - 3^{10} \cdot 5 = 0 \Leftrightarrow x = 5 \cdot 3^{10} \\ \downarrow \text{sau} \\ x + 3^{10} \cdot 5 = 0 \Leftrightarrow x = -5 \cdot 3^{10} \end{cases} \Leftrightarrow$
 $\Leftrightarrow \underline{x \in \{-5 \cdot 3^{10}; 5 \cdot 3^{10}\}}$

18. $\underline{x^2 = \sqrt{3} - 2}$
semnal $(\sqrt{3} - 2)$? $\left. \begin{array}{l} \sqrt{3} - 2 = \sqrt{3} - \sqrt{4} < 0 \Rightarrow \sqrt{3} - 2 < 0 \\ \sqrt{3} < \sqrt{4} \end{array} \right\} \Rightarrow \underline{x \in \emptyset}$

19. $\frac{x^2+4x}{8} = \frac{2x+3}{4} \quad | \cdot 8 \Leftrightarrow x^2+4x = 2(2x+3) \Leftrightarrow x^2+4x = 4x+6 \Leftrightarrow x^2+4x-4x-6=0 \Leftrightarrow$
 $\Leftrightarrow x^2-(\sqrt{6})^2=0 \Leftrightarrow (x-\sqrt{6})(x+\sqrt{6})=0 \Leftrightarrow \begin{cases} x-\sqrt{6}=0 \Rightarrow x=\sqrt{6} \\ x+\sqrt{6}=0 \Rightarrow x=-\sqrt{6} \end{cases} \Leftrightarrow$
 $\Leftrightarrow x \in \{-\sqrt{6}; \sqrt{6}\}$

20. $\frac{x^2}{\sqrt{11}-7} = \sqrt{11}+7 \Leftrightarrow x^2 = (\sqrt{11}-7)(\sqrt{11}+7) \Leftrightarrow x^2 = (\sqrt{11})^2 - 7^2 \Leftrightarrow x^2 = 11 - 49 \Leftrightarrow x^2 = -38 < 0$
 $(a-b)(a+b) = a^2 - b^2 \Leftrightarrow x \in \emptyset$

21. $\frac{1}{4-x} + \frac{1}{4+x} = \frac{1}{2} \quad (x \neq 4; x \neq -4) \Leftrightarrow \frac{1}{4-x} + \frac{1}{4+x} = \frac{1}{2} \Leftrightarrow \frac{4+x+4-x}{(4-x)(4+x)} = \frac{1}{2} \Leftrightarrow$
 $\Leftrightarrow \frac{8}{4^2-x^2} = \frac{1}{2} \Leftrightarrow 8 \cdot 2 = 16 - x^2 \Leftrightarrow x^2 = 16 - 16 \Leftrightarrow x^2 = 0 \Rightarrow x = 0$

22. $\frac{(x+3\sqrt{3})(x+2\sqrt{3})}{\sqrt{75}} = 18 + \sqrt{75}x \Leftrightarrow x^2 + 2\sqrt{3}x + 3\sqrt{3}x + 3\sqrt{3} \cdot 2\sqrt{3} = 18 + 5\sqrt{3}x \Leftrightarrow$
 $\Leftrightarrow x^2 + 5\sqrt{3}x + 6 \cdot 3 = 18 + 5\sqrt{3}x \Leftrightarrow x^2 + 5\sqrt{3}x - 5\sqrt{3}x = 18 - 18$
 $\Leftrightarrow x^2 = 0 \Leftrightarrow x = 0 \Leftrightarrow x \in \{0\}$

$\sqrt{75} = \sqrt{3 \cdot 25} = 5\sqrt{3}$
 $\sqrt{3} \cdot \sqrt{3} = 3$