

Mind Generation
Centru de Matematica si Informatica

Operatii cu intervale
Clasa a VIII-a

1. Scrieti ca interval multimele:

a) $A = \{x \in \mathbb{R} \mid |2x-3| < 7\}$

$$\begin{aligned} |2x-3| < 7 &\Leftrightarrow -7 < 2x-3 < 7 \quad | +3 \Leftrightarrow \\ \Leftrightarrow -4 < 2x < 10 \quad | :2 > 0 &\Leftrightarrow -2 < x < 5 \Leftrightarrow \\ \Leftrightarrow x \in (-2, 5) &\Rightarrow A = (-2, 5) \end{aligned}$$

b) $B = \{x \in \mathbb{R} \mid 2 < \frac{1-3x}{5} \leq 6\}$

$$\begin{aligned} 2 < \frac{1-3x}{5} \leq 6 \quad | \cdot 5 > 0 &\Leftrightarrow 10 < 1-3x \leq 30 \quad | -1 \\ \Leftrightarrow 9 < -3x \leq 29 \quad | : (-3) < 0 &\Leftrightarrow \\ \Leftrightarrow -3 > x \geq \frac{-29}{3} &\quad \left\{ \begin{array}{l} \text{se schimbă} \\ \text{sensul inegalităților} \\ \text{deoarece } -3 < 0 \end{array} \right. \\ \Leftrightarrow \frac{-29}{3} \leq x < -3 &\Leftrightarrow \\ \Leftrightarrow x \in \left[\frac{-29}{3}; -3 \right) &\Rightarrow B = \left[\frac{-29}{3}; -3 \right) \end{aligned}$$

c) $C = \{x \in \mathbb{R} \mid 2x+4 \in [1;6]\}$

$$\begin{aligned} 2x+4 \in [1;6] &\Leftrightarrow 1 \leq 2x+4 \leq 6 \quad | -4 \Leftrightarrow \\ \Leftrightarrow -3 \leq 2x \leq 2 \quad | :2 > 0 &\Leftrightarrow -\frac{3}{2} \leq x \leq 1 \Leftrightarrow \\ \Leftrightarrow x \in \left[-\frac{3}{2}; 1 \right] & \\ \Rightarrow C = \left[-\frac{3}{2}; 1 \right] & \end{aligned}$$

d) $D = \{x \in \mathbb{R} \mid |2x-\frac{1}{3}| < 0, (3)\}$

$$\begin{aligned} |2x-\frac{1}{3}| < 0, (3) &\quad \left. \begin{array}{l} |2x-\frac{1}{3}| < 0, (3) \\ 0, (3) = \frac{3}{9} = \frac{1}{3} \end{array} \right\} \Rightarrow |2x-\frac{1}{3}| < \frac{1}{3} \Leftrightarrow \\ \Leftrightarrow -\frac{1}{3} < 2x-\frac{1}{3} < \frac{1}{3} &\quad | +\frac{1}{3} \Leftrightarrow \\ \Leftrightarrow 0 < 2x < \frac{2}{3} \quad | :2 &\Leftrightarrow 0 < x < \frac{1}{3} \\ \Rightarrow x \in (0, \frac{1}{3}) &\Rightarrow D = (0, \frac{1}{3}) \end{aligned}$$


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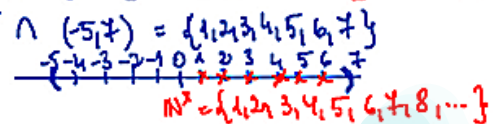
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2. Efectuati:

a) $\{3,7\} \cup \{3,7\} = \{3,7\}$



b) $\mathbb{N}^* \cap (-5,7) = \{1,2,3,4,5,6,7\}$

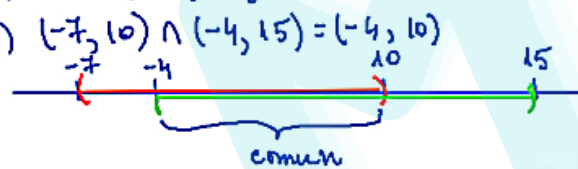


$\mathbb{N}^* = \{1,2,3,4,5,6,7,8, \dots\}$

c) $(-5,4) \cap \{4\} = \emptyset$

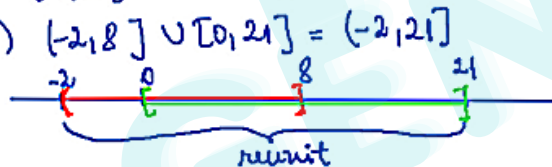
d) $(-2,5) \setminus \{2\} = (-2,5)$

e) $(-7,10) \cap (-4,15) = (-4,10)$

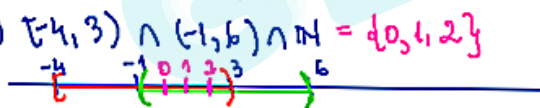


f) $(-4,6] \cap \mathbb{Z} = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

g) $(-2,8] \cup [0,21] = (-2,21]$



h) $(-4,3) \cap (-1,6) \cap \mathbb{N} = \{0,1,2\}$



3. Stabiliti valoarea de adevar a propozitiilor:

a) $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{29 \cdot 30} \in [0,3); 0,5]$

Calculăm suma, pe baza relației:

$$\frac{1}{n-1} - \frac{1}{n} = \frac{1}{n(n-1)}$$

$$\Rightarrow \frac{1}{2 \cdot 3} = \frac{1}{2} - \frac{1}{3}$$

$$\frac{1}{3 \cdot 4} = \frac{1}{3} - \frac{1}{4}$$

$$\dots$$

$$\frac{1}{28 \cdot 29} = \frac{1}{28} - \frac{1}{29}$$

$$\frac{1}{29 \cdot 30} = \frac{1}{29} - \frac{1}{30}$$

Adunăm

$$\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{29 \cdot 30} = \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{28} - \frac{1}{29} + \frac{1}{29} - \frac{1}{30} =$$

$$= \frac{15}{2} - \frac{1}{30} = \frac{15 \cdot 1}{30} = \frac{14}{30}$$

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de arătat că:
 $0,3 = \frac{3}{10} = \frac{1}{3} ; 0,5 = \frac{1}{2}$

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$$\Leftrightarrow \frac{19}{3} \leq \frac{14}{30} \leq \frac{15}{2} \Leftrightarrow \frac{10}{30} \leq \frac{14}{30} \leq \frac{15}{30} \quad | \cdot 30 \Leftrightarrow$$

$$\Leftrightarrow 10 \leq 14 \leq 15 \text{ "A"}$$

$$b) \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{99 \cdot 100} \in \left[\frac{51}{100}; 1 \frac{1}{10} \right]$$

$$\frac{1}{1 \cdot 2} = \frac{1}{1} - \frac{1}{2}$$

$$\frac{1}{2 \cdot 3} = \frac{1}{2} - \frac{1}{3}$$

$$\frac{1}{3 \cdot 4} = \frac{1}{3} - \frac{1}{4}$$

$$\dots$$

$$\frac{1}{98 \cdot 99} = \frac{1}{98} - \frac{1}{99}$$

$$\frac{1}{99 \cdot 100} = \frac{1}{99} - \frac{1}{100} \text{ Adunăm}$$

$$S = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{98} - \frac{1}{99} + \frac{1}{99} - \frac{1}{100} =$$

$$= \frac{100}{1} - \frac{1}{100} = \frac{100-1}{100} = \frac{99}{100} \left. \vphantom{\frac{100}{1}} \right\} \Rightarrow \text{de arătat că:}$$

$$1 \frac{1}{10} = \frac{1 \cdot 10 + 1}{10} = \frac{11}{10} \quad \frac{99}{100} \in \left[\frac{51}{100}; \frac{11}{10} \right] \Leftrightarrow$$

$$\Leftrightarrow \frac{51}{100} \leq \frac{99}{100} \leq \frac{11}{10} \Leftrightarrow$$

$$\Leftrightarrow \frac{51}{100} \leq \frac{99}{100} \leq \frac{110}{100} \text{ ("A")}$$

4. Determinati: $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$

pentru fiecare din cazurile:

$$a) A = \left\{ x \in \mathbb{R} \mid -9 < \frac{5x+12}{2} < 16 \right\}; B = \left\{ x \in \mathbb{R} \mid -8 < \frac{7x+12}{2} < 27 \right\}$$

$$-9 < \frac{5x+12}{2} < 16 \quad | \cdot 2 > 0 \Leftrightarrow -18 < 5x+12 < 32 \quad | -12 \Leftrightarrow$$

$$\Leftrightarrow -30 < 5x < 20 \quad | :5 > 0 \Leftrightarrow -6 < x < 4 \Rightarrow x \in (-6, 4) \Rightarrow$$

$$\Rightarrow A = (-6, 4)$$

$$-8 < \frac{7x+12}{2} < 27 \quad | \cdot 2 > 0 \Leftrightarrow -16 < 7x+12 < 54 \quad | -12 \Leftrightarrow$$

$$\Leftrightarrow -28 < 7x < 42 \quad | :7 > 0 \Leftrightarrow$$

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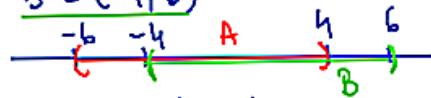
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$$\Leftrightarrow -4 < x < 6 \Leftrightarrow x \in (-4, 6) \Rightarrow B = (-4, 6)$$

$$A = (-6, 4)$$

$$B = (-4, 6)$$



$$\Rightarrow A \cup B = (-6, 6); A \cap B = (-4, 4)$$

$$A \setminus B = (-6, -4]; B \setminus A = [4, 6)$$

$$6) A = \{x \in \mathbb{R} \mid |2x-7| \leq 3\}; B = \{x \in \mathbb{R} \mid \left| \frac{4x-3}{3} \right| \leq 5\}$$

$$|2x-7| \leq 3 \Leftrightarrow -3 \leq 2x-7 \leq 3 \mid +7 \Leftrightarrow -4 \leq 2x \leq 10 \mid :2$$

$$\Leftrightarrow -2 \leq x \leq 5 \Rightarrow A = [-2; 5]$$

$$\left| \frac{4x-3}{3} \right| \leq 5 \Leftrightarrow -5 \leq \frac{4x-3}{3} \leq 5 \mid \cdot 3 \geq 0 \Leftrightarrow$$

$$\Leftrightarrow -15 \leq 4x-3 \leq 15 \mid +3 \Leftrightarrow -12 \leq 4x \leq 18 \mid :4 \Leftrightarrow$$

$$\Leftrightarrow -3 \leq x \leq \frac{18}{4} \Leftrightarrow -3 \leq x \leq \frac{9}{2} \Leftrightarrow$$

$$\Leftrightarrow B = \left[-3; \frac{9}{2}\right]$$

$$\Rightarrow A \cup B = \left[-3; \frac{9}{2}\right]; A \cap B = \left[-2; \frac{9}{2}\right]; A \setminus B = \left(\frac{9}{2}; 5\right]; B \setminus A = [-3; -2)$$

$$c) A = \{x \in \mathbb{R} \mid |2x-3| \leq 11\}; B = \{x \in \mathbb{Z} \mid -1 < \frac{3x+7}{8} < 2\}$$

$$|2x-3| \leq 11 \Leftrightarrow -11 \leq 2x-3 \leq 11 \mid +3 \Leftrightarrow -8 \leq 2x \leq 14 \mid :2 > 0$$

$$\Leftrightarrow -4 \leq x \leq 7 \Rightarrow A = [-4; 7]$$

$$-1 < \frac{3x+7}{8} < 2 \mid \cdot 8 > 0 \Leftrightarrow -8 < 3x+7 < 16 \mid -7 \Leftrightarrow$$

$$\Leftrightarrow -15 < 3x < 9 \mid :3 > 0 \Leftrightarrow -5 < x < 3 \Bigg\} \Rightarrow x \in \mathbb{Z}$$

$$\Rightarrow x \in \{-4, -3, -2, -1, 0, 1, 2\} \Rightarrow B = \{-4, -3, -2, -1, 0, 1, 2\}$$

$$\Rightarrow A \cup B = [-4; 7]; A \cap B = \{-4, -3, -2, -1, 0, 1, 2\}$$

$$A \setminus B = (-4; -3) \cup (-3; -2) \cup (-2; -1) \cup (-1; 0) \cup (0; 1) \cup (1; 2) \cup (2; 7]$$

$$B \setminus A = \emptyset$$

$$5. \text{ Efectuati: } \underbrace{(-4; 4] \cap [-3; 8)}_{= [-3; 4]} \cup \{-3, -2, 2, 4, 5, 6\} = [-3; 4] \cup \{5, 6\}$$

