Mind Generation SRL
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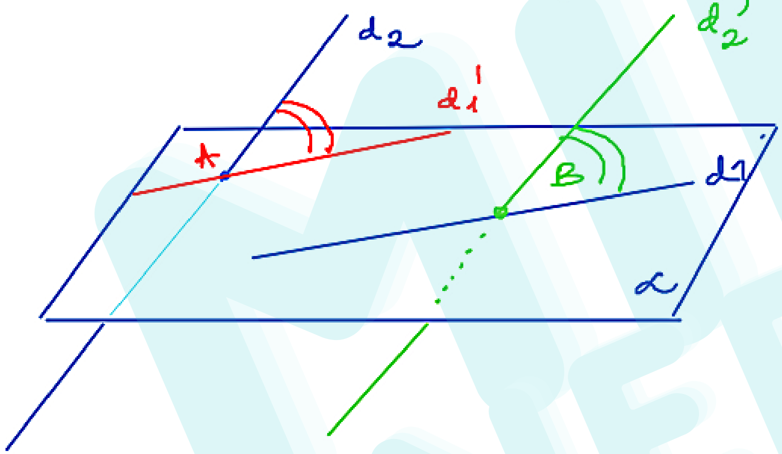
Mind Generation
Centru de Matematica si Informatica

Recapitulare geometrie cls a VIII-a
Unghiuri in spatiu, distante, Teorema celor 3 perpendiculare

13.02.202

(1) Unghiul dintre 2 drepte în spațiu



$d_1 \subset \alpha$

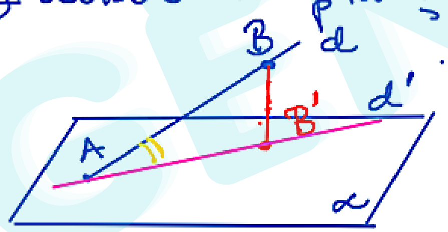
$d_1 \cap d_2 = \emptyset$

$d_1' \parallel d_1 \Rightarrow d_1' \cap d_2 = \{A\}$

$d_2' \parallel d_2 \Rightarrow d_2' \cap d_1' = \{B\}$

$\widehat{(d_1, d_2)} = \widehat{(d_1', d_2)} = \widehat{(d_1, d_2')}$

(2) ∡ dintre dreaptă și un plan



$d \not\subset \alpha$; $d \cap \alpha = \{A\}$

$\forall B \in d$; $B' = pr_\alpha B \Rightarrow AB' = pr_\alpha \underset{AB}{\overset{d}{\parallel}} \Rightarrow$

$(BB' \perp \alpha)$ $\Rightarrow \widehat{(d, \alpha)} = \widehat{BAB'} =$
$= \widehat{(d, pr_\alpha d)}$

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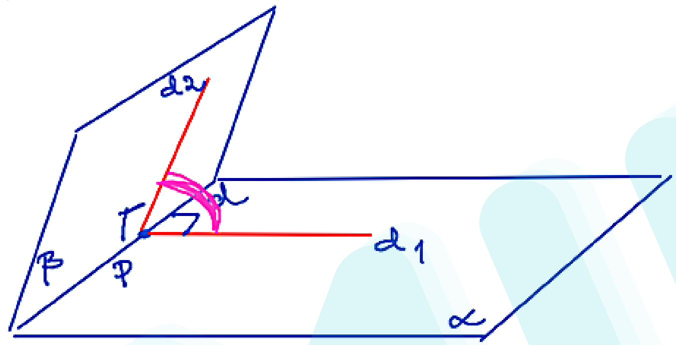
3. ∡ diedru = $\sphericalangle(\alpha, \beta)$



Dc. $\alpha \parallel \beta \Rightarrow \alpha \cap \beta = \emptyset$

Dc. $\alpha \nparallel \beta \Rightarrow \alpha \cap \beta = d$

a). $\alpha \cap \beta = d$

b) Considerăm $P \in d$. ($\forall$ pct de pe d)

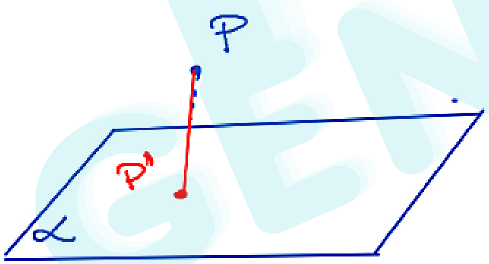
c) $d_1 \perp d$, $d_1 \subset \alpha$, $P \in d_1$

$d_2 \perp d$, $d_2 \subset \beta$, $P \in d_2$

$\Rightarrow \widehat{(\alpha, \beta)} = \widehat{(d_1, d_2)}$

Obs: În orice problemă {pas 1. identificăm ∡-ul diedru cf. a), b), c)
pas 2. Calculăm ∡-ul diedru

4.



$P \notin \alpha$

$d(P, \alpha) = ?$

→ ducem $PP' \perp \alpha$, $P' \in \alpha \Rightarrow d(P, \alpha) = PP'$

$P' = pr_\alpha P$

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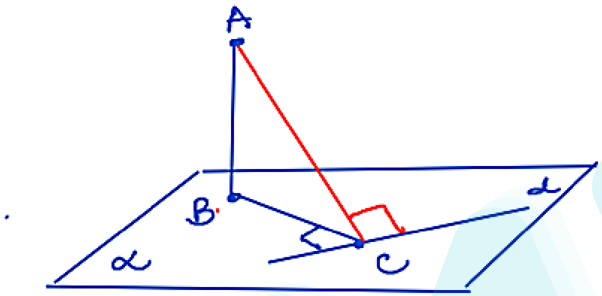
5 T. celor 3 ⊥

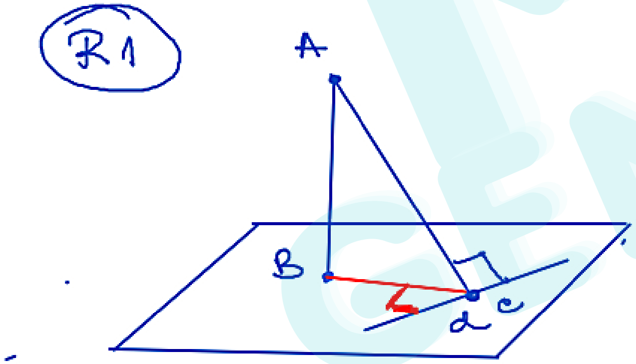


$$\left.\begin{array}{l} d \subset \alpha \\ AB \perp \alpha,\ A \notin \alpha,\ B \in \alpha \\ \text{Ducem } BC \perp d,\ BC \subset \alpha \end{array}\right\} \xRightarrow{T.3\perp}$$

$$\Rightarrow \boxed{AC \perp d}$$

2 Reciproce:

R1



$$\left.\begin{array}{l} d \subset \alpha \\ AB \perp \alpha,\ A \notin \alpha,\ B \in \alpha \\ AC \perp d \end{array}\right\} \xRightarrow{R.3\perp} \boxed{BC \perp d}$$

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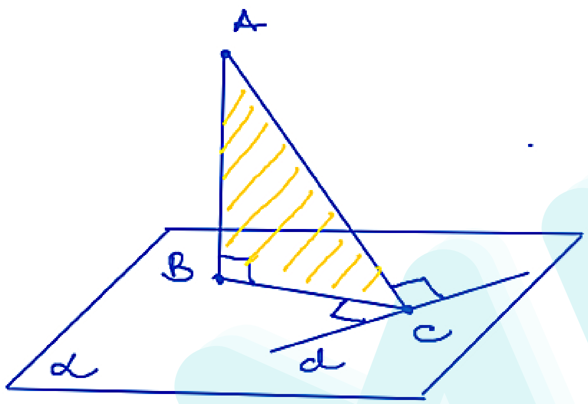
R2



$d \subset \alpha$
AB ; $A \notin \alpha$, $B \in \alpha$
$AC \perp d$; $C \in d$
$BC \perp d$
$AB \perp BC \subset \alpha$

$\Big\} \xRightarrow{R2.3\perp} \boxed{AB \perp \alpha}$

$$\left.\begin{matrix} d \perp BC \\ d \perp AC \end{matrix}\right\} \Rightarrow \left.\begin{matrix} d \perp (ABC) \\ AB \subset (ABC) \end{matrix}\right\} \Rightarrow d \perp AB \Rightarrow \left.\begin{matrix} AB \perp d \\ AB \perp BC \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow AB \perp (d, BC) = \alpha$$

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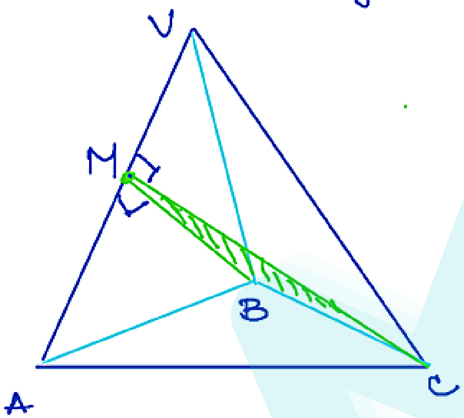
Probleme

① Piramidă $\triangle$ regulată:



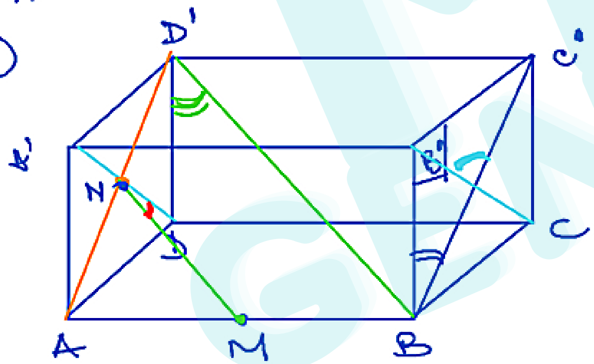
$(\widehat{VA, BC}) = ?$

$\triangle VAB \equiv \triangle VAC$

$\left.\begin{array}{l} CM \perp VA \\ BM \perp VA \end{array}\right\} \Rightarrow \left.\begin{array}{l} VA \perp (MBC) \\ BC \subset (MBC) \end{array}\right\} \Rightarrow \boxed{VA \perp BC}$

$\Rightarrow (\widehat{VA, BC}) = 90^\circ$

②



a) $(\widehat{AA', BC'}) = (\widehat{BB', BC'}) = \widehat{B'BC'}$

$AA' \parallel BB'$

Dacă este cub $\Rightarrow 45^\circ$

b) $(\widehat{A'D, BC'}) = (\widehat{B'C, BC'})$

$B'C \parallel A'D$

Dacă este cub $\Rightarrow 90^\circ$

c) $(\widehat{AA', D'B}) = \widehat{DD'B}$ $(AA' \parallel DD')$

a) $(\widehat{D'B, B'C})$

$\left.\begin{array}{l} M = \text{mijl } [AB] \\ N = AB' \cap A'D \end{array}\right\} \Rightarrow$

$\Rightarrow NM = \text{l.m. în } \triangle ABD'$

$\Rightarrow NM \parallel D'B$

$B'C \parallel A'D$ $\Rightarrow \widehat{MND}$

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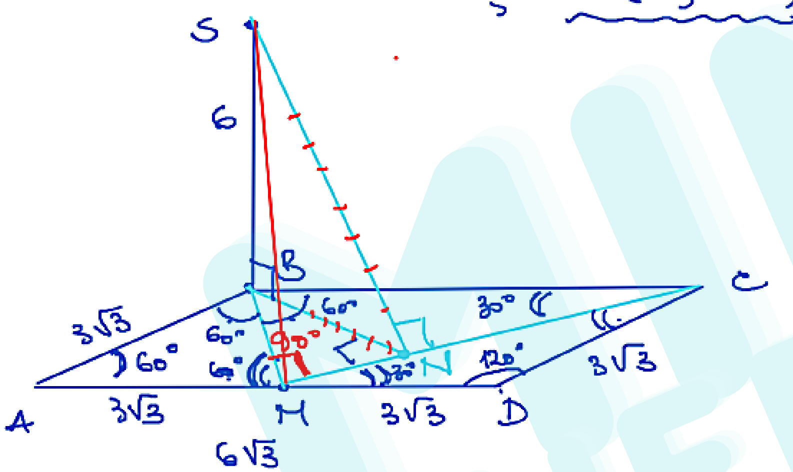
3. În paralelogramul ABCD cu $\hat{A} = 60°$, $AB = 3\sqrt{3}$ cm, $AD = 6\sqrt{3}$ cm, se consideră M mijlocul (AD) și perpendiculara pe pl. (ABC) în pct. B, BS = 6 cm. Calculați $d(S, MC) = ?$



T.3⊥ !

D: Ducem $BN \perp MC$
$SB \perp (ABC)$
$MC \subset (ABC)$
$\Big\} \overset{T.3\perp}{\Longrightarrow}$

$\Rightarrow SN \perp MC \Rightarrow d(S, MC) = \underline{SN}$

$M = \text{mijl } |AD| \Rightarrow AM = \dfrac{AD}{2} = \dfrac{6\sqrt{3}}{2} = \underline{3\sqrt{3} \text{ cm}}$

$\Rightarrow \Delta ABM \begin{cases} \widehat{BAM} = 60° \\ AB = AM = 3\sqrt{3} \text{ cm} \end{cases} \Rightarrow$ echilateral $\Rightarrow BM = 3\sqrt{3}$ cm $\Rightarrow$
$\Rightarrow \widehat{AMB} = 60°$

$\Delta DMC \begin{cases} MD = DC = 3\sqrt{3} \text{ cm} \\ \widehat{MDC} = 180° - 60° = 120° \end{cases} \Bigg\} \Rightarrow \widehat{DMC} = \widehat{DCM} = \dfrac{180° - 120°}{2} = \underline{30°} \Rightarrow$
$\Rightarrow \widehat{BMC} = 90° \Rightarrow BM \perp MC \overset{T.3\perp}{\Longrightarrow} \underline{SM \perp MC}$

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$\Delta SBM \begin{cases} \widehat{SBM} = 90^\circ \\ SB = 6\text{cm} \\ BM = 3\sqrt{3}\text{ cm} \end{cases} \xRightarrow{T.P.} SM^2 = SB^2 + BM^2 \Rightarrow SM^2 = 6^2 + (3\sqrt{3})^2 =$

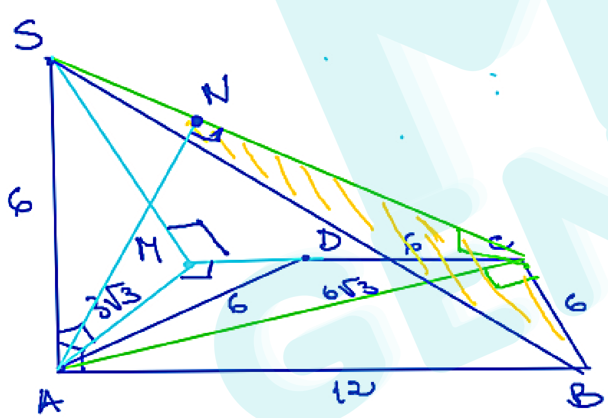
$= 36 + 27 = 63$

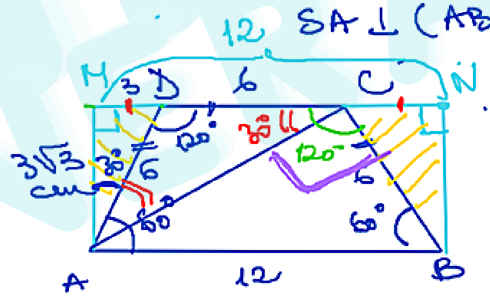
$\Rightarrow SM = \sqrt{63} = \underline{3\sqrt{7}\text{ cm}}$

4 În pl. trapezului ABCD, $AB \parallel CD$, $AB = 12$cm, $BC = CD = DA = 6$ cm. Se ridică perpendiculara $SA = 6$ cm. Calculați:

a) $d(S, CD) = ?$ b) $d(S, BC)$; c) $d(A, (SBC))$; d) $d(A, (SCD))$

a) Ducem $AM \perp DC$
$DC \subset (ABC)$
$SA \perp (ABC)$
$\xRightarrow{T.3\perp} SM \perp MC \Rightarrow$
$\Rightarrow d(S, DC) = \underline{SM}$





$MD = \dfrac{AB - DC}{2} = \dfrac{12 - 6}{2} = 3$ cm

$\Delta AMD \begin{cases} \widehat{AMD} = 90^\circ \\ AD = 6\text{ cm} \\ MD = 3\text{ cm} \end{cases} \xRightarrow{T.P.} AD^2 = AM^2 + MD^2 \Rightarrow$

$\Rightarrow AM^2 = 36 - 9 = 27 \Rightarrow \underline{AM = 3\sqrt{3}\text{ cm}}$

ΔSAM: T.P. $SM^2 = SA^2 + AM^2 = 36 + 27 = 63 \Rightarrow SM = 3\sqrt{7}$

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$\Delta MAD \begin{cases} \text{dreptunghic} \\ MD = 3cm \\ AD = 6cm \end{cases} \xrightarrow{R.T.30^\circ} \widehat{MAD} = 30^\circ \Rightarrow \widehat{DAB} = \widehat{CBA} = 60^\circ$ și $\widehat{ADC} = \widehat{BCD} = 120^\circ$.

$\Delta DAC \begin{cases} \widehat{ADC} = 120^\circ \\ DA = DC = 6cm \end{cases} \Rightarrow \widehat{DCA} = \widehat{DAC} = \frac{180^\circ - 120^\circ}{2} = \underline{\underline{30^\circ}}$

$\Rightarrow \widehat{ACD} = 120^\circ - 30^\circ = 90^\circ \Rightarrow$

$\left.\begin{array}{l} \Rightarrow AC \perp CB \\ SA \perp (ABC) \\ CB \subset (ABC) \end{array}\right\} \xrightarrow{T.3\perp} SC \perp BC$

$\Delta ABC: \sin \widehat{ABC} = \frac{AC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AC}{12} \Rightarrow AC = \frac{\cancel{12}^{6} \cdot \sqrt{3}}{\cancel{2}_{1}} = \underline{\underline{6\sqrt{3}\ cm}}$

$\widehat{ABC} = 60^\circ$

$\Delta SAC \begin{cases} SA = 6cm \\ AC = 6\sqrt{3}\ cm \\ \widehat{SAC} = 90^\circ \end{cases} \xrightarrow{T.P.} SC^2 = SA^2 + AC^2 = 6^2 + 6^2 \cdot 3 = 6^2(1+3) = 6^2 \cdot 4 \Rightarrow$

$\Rightarrow SC = \sqrt{6^2 \cdot 4} = 6 \cdot 2 = \underline{\underline{12\ cm}}$

c) $\left.\begin{array}{l} \text{Ducem } AN \perp SC \\ BC \subset (SBC) \\ NC \perp BC \\ AC \perp BC \end{array}\right\} \xrightarrow{R.2T.3\perp} AN \perp (SBC) \Rightarrow d(A, (SBC)) = AN$

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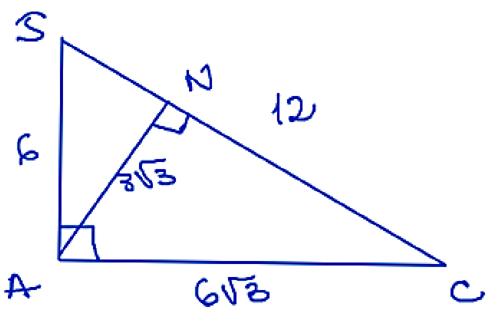
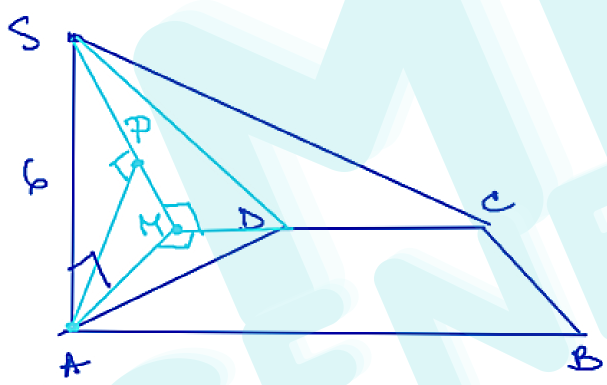


$$AN = \frac{SA \cdot AC}{SC} = \frac{\cancel{6}^{3} \cdot \cancel{6}\sqrt{3}}{\cancel{12}\ \cancel{2}\ 1} = 3\sqrt{3}\text{ cm} \Rightarrow d(A, (SBC)) = 3\sqrt{3}\text{ cm}$$

d)



$$\begin{matrix}(SMC)\\ \parallel \\ d(A, (\underline{SDC})) = ?\end{matrix}$$

$$\left.\begin{aligned}&\text{Ducem } AP \perp SM\\ &MC \subset (SDC)\\ &AM \perp MC\\ &SM \perp MC\end{aligned}\right\} \Rightarrow AP \perp (SMC) \Rightarrow \Rightarrow d(A, (SDC)) = \underline{\underline{AP}}$$

$$\Delta SAM \left\{\begin{aligned}&\widehat{SAM} = 90^\circ\\ &AM = 3\sqrt{3}\text{ cm}\\ &SM = 3\sqrt{7}\text{ cm}\end{aligned}\right\} \Rightarrow AP = \frac{SA \cdot AM}{SM} =$$

$$\sqrt{7})$$

$$= \frac{6 \cdot \cancel{3}\sqrt{3}}{\cancel{3}\sqrt{7}} = \frac{6\sqrt{21}}{7}\ \underline{\underline{\text{cm}}} \Rightarrow d(A, (SDC)) = \frac{6\sqrt{21}}{7}\text{ cm}$$

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Temă: (1) Triunghiul ABC are latura AB de 8 cm. Pe perpendiculara în A pe planul (ABC) se ia un pct. D, a.î. $DC \perp BC$. Determinați raza cercului circumscris Δului ABC.

(2) Demonstrați că în paralelipipedul dreptunghic $ABCDA'B'C'D'$ proiecția punctului B pe planul (ACB') este ortocentrul Δului ACB'