

Mind Generation
Centru de Matematica si Informatica

Exercitii cu radicali - cls. a VII-a
~ Exercitii selectate din Testele de EN aparute pentru EN 2020

1. Se considera numerele reale: $a = \frac{4}{\sqrt{20} + \sqrt{12}} + \sqrt{3} - 1$; $b = \frac{6}{2\sqrt{8} - \sqrt{20}} - \sqrt{8} + 1$

- a). Calculati valorile numerelor reale a si b
b). Calculati media aritmetica si media geometrica ale numerelor a si b

a). ~~Pz.~~ calculul a și b, mai întâi scoatem de sub radical:

$$\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}; \quad \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}; \quad \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2} \Rightarrow a = \frac{4}{2\sqrt{5} + 2\sqrt{3}} + \sqrt{3} - 1. \text{ Vom rationaliza:}$$

$$a = \frac{\frac{\sqrt{5} - \sqrt{3}}{4}}{2(\sqrt{5} + \sqrt{3})} + \sqrt{3} - 1 = \frac{4(\sqrt{5} - \sqrt{3})}{2 \cdot (5 - 3)} + \sqrt{3} - 1 =$$
$$= \frac{4(\sqrt{5} - \sqrt{3})}{2 \cdot 2} + \sqrt{3} - 1 = \sqrt{5} - \sqrt{3} + \sqrt{3} - 1 = \sqrt{5} - 1$$

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a). Calculati valorile numerelor reale a si b

b). Calculati media aritmetica si media geometrica ale numerelor a si b

$$\begin{aligned} b &= \frac{6}{2 \cdot 2\sqrt{2} - 2\sqrt{5}} - 2\sqrt{2} + 1 = \frac{\frac{2(2+\sqrt{3})}{6} \cdot 3}{\cancel{2}(2\sqrt{2}-\sqrt{5})} - 2\sqrt{2} + 1 = \\ &= \frac{3(2\sqrt{2} + \sqrt{5})}{(2\sqrt{2})^2 - (\sqrt{5})^2} - 2\sqrt{2} + 1 = \frac{3(2\sqrt{2} + \sqrt{5})}{8-5} - 2\sqrt{2} + 1 = \\ &\stackrel{1}{=} \frac{\cancel{3}(2\sqrt{2} + \sqrt{5})}{\cancel{3}1} - 2\sqrt{2} + 1 = \cancel{2}\sqrt{2} + \sqrt{5} - \cancel{2}\sqrt{2} + 1 = \sqrt{5} + 1 \end{aligned}$$

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- a). Calculati valorile numerelor reale a si b
b). Calculati media aritmetica si media geometrica ale numerelor a si b

a) $a = \sqrt{5} - 1$

$b = \sqrt{5} + 1$

$$m_a = \frac{a+b}{2} = \frac{\sqrt{5} - 1 + \sqrt{5} + 1}{2} = \frac{2\sqrt{5}}{2} = \underline{\underline{\sqrt{5}}}$$

$$m_g = \sqrt{a \cdot b} = \sqrt{(\sqrt{5} - 1)(\sqrt{5} + 1)} = \sqrt{(\sqrt{5})^2 - 1^2} =$$

$$= \sqrt{5 - 1} = \sqrt{4} = \underline{\underline{2}}$$

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2. Se considera numerele: $a = \sqrt{(\sqrt{2} - \sqrt{27})^2}$ si $b = \sqrt{27} + (\sqrt{5} - \sqrt{3})^2 - \left(\frac{10\sqrt{3}}{\sqrt{5}} - \frac{12\sqrt{5}}{\sqrt{3}}\right) + \sqrt{2}(1 - 4\sqrt{2})$

- a). Calculati valorile numerelor a si b
b). Aratati ca numarul real

$$x = \frac{1}{a} + \frac{1}{b} - \frac{48}{25\sqrt{3}} \quad \text{verifica inegalitatile: } -\frac{4}{5} < x < -0,6$$

a) Vom folosi definitia modului:

$$|x| = \begin{cases} x, & \text{daca } x \geq 0 \\ -x, & \text{daca } x < 0 \end{cases} \quad (1)$$

si formulele:

$$\sqrt{x^2} = |x| \quad (2)$$

$$(a-b)^2 = a^2 - 2ab + b^2 \quad (3)$$

$$\text{Deci din (2)} \Rightarrow a = \sqrt{(\sqrt{2} - \sqrt{27})^2} = |\sqrt{2} - \sqrt{27}| \left. \begin{array}{l} \text{din (1)} \\ \text{cum } \sqrt{2} - \sqrt{27} < 0 \end{array} \right\} \Rightarrow a = -(\sqrt{2} - \sqrt{27}) = \sqrt{27} - \sqrt{2} = 3\sqrt{3} - \sqrt{2}$$

$$b = 3\sqrt{3} + [(\sqrt{5})^2 - 2 \cdot \sqrt{5} \cdot \sqrt{3} + (\sqrt{3})^2]^2 - \frac{10\sqrt{3}}{\sqrt{5}} + \frac{12\sqrt{5}}{\sqrt{3}} + \sqrt{2} - \sqrt{2} \cdot 4\sqrt{2} =$$

($\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$)

$$= 3\sqrt{3} + (5 - 2\sqrt{15} + 3) - \frac{2\sqrt{15}}{5} + \frac{4\sqrt{15}}{3} + \sqrt{2} - 4 \cdot 2 = 3\sqrt{3} + 8 - 2\sqrt{15} - 2\sqrt{15} + 4\sqrt{15} + \sqrt{2} - 8 =$$

$$= 3\sqrt{3} + \sqrt{2}$$

$$e) \quad x = \frac{1}{a} + \frac{1}{b} - \frac{48}{25\sqrt{3}} = \frac{3\sqrt{3} + \sqrt{2}}{3\sqrt{3} - \sqrt{2}} + \frac{3\sqrt{3} - \sqrt{2}}{3\sqrt{3} + \sqrt{2}} - \frac{\sqrt{3}}{25} = \frac{3\sqrt{3} + \sqrt{2}}{(3\sqrt{3})^2 - (\sqrt{2})^2} + \frac{3\sqrt{3} - \sqrt{2}}{(3\sqrt{3})^2 - (\sqrt{2})^2} -$$

Folosim: $(a-b)(a+b) = a^2 - b^2$ (pt. rationalizarea numitorilor)

$$\frac{16\sqrt{3}}{25} = \frac{3\sqrt{3} + \sqrt{2}}{27 - 2} + \frac{3\sqrt{3} - \sqrt{2}}{27 - 2} - \frac{16\sqrt{3}}{25} = \frac{3\sqrt{3} + \sqrt{2} + 3\sqrt{3} - \sqrt{2} - 16\sqrt{3}}{25} = \frac{-10\sqrt{3}}{25} = -\frac{2\sqrt{3}}{5}$$

$$\Rightarrow x = \sqrt{3}$$

$$\text{De dem. că: } -\frac{4}{5} < -\frac{2\sqrt{3}}{5} < -0,6 \quad | \cdot (-1) \Leftrightarrow \frac{4}{5} > \frac{2\sqrt{3}}{5} > 0,6 \Leftrightarrow$$

$$\Leftrightarrow \frac{4}{5} > \frac{2\sqrt{3}}{5} > \frac{3}{5} \Leftrightarrow \frac{4}{5} > \frac{2\sqrt{3}}{5} > \frac{3}{5} \quad | \cdot 5 \Leftrightarrow 4 > 2\sqrt{3} > 3 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{16} > \sqrt{4 \cdot 3} > \sqrt{9} \Leftrightarrow \sqrt{16} > \sqrt{12} > \sqrt{9} \quad \text{Adevărat} \Rightarrow$$

$$\Rightarrow -\frac{4}{5} < x < -0,6 \quad \text{Adevărat}$$

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3. Se considera numerele: $a = 2\sqrt{6} \left(\frac{3}{\sqrt{2}} - \frac{5}{\sqrt{3}} \right) + \frac{18}{3\sqrt{2}+2\sqrt{3}}$

$$b = \frac{10}{4\sqrt{2}+3\sqrt{3}} - 3 \sqrt{(2\sqrt{3}-3\sqrt{2})^2} + 1$$

Calculati: $(a-b)^{2021}$

$$a = 2\sqrt{\frac{\sqrt{2}}{6}} \cdot \frac{3}{\sqrt{2}} - 2\sqrt{\frac{\sqrt{2}}{6}} \cdot \frac{5}{\sqrt{3}} + \frac{18 \cdot \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= 6\sqrt{3} - 10\sqrt{2} + \frac{18(3\sqrt{2}-2\sqrt{3})}{9 \cdot 2 - 4 \cdot 3} = 6\sqrt{3} - 10\sqrt{2} + \frac{18(3\sqrt{2}-2\sqrt{3})}{81} =$$

$$= \cancel{6\sqrt{3}} - 10\sqrt{2} + 9\sqrt{2} - \cancel{6\sqrt{3}} = \underline{\underline{-\sqrt{2}}}$$

$$b = \frac{\frac{4\sqrt{2}-3\sqrt{3}}{10}}{4\sqrt{2}+3\sqrt{3}} - 3 \sqrt{(2\sqrt{3}-3\sqrt{2})^2} + 1 = \frac{10(4\sqrt{2}-3\sqrt{3})}{(4\sqrt{2})^2 - (3\sqrt{3})^2} + 3|2\sqrt{3}-3\sqrt{2}| + 1 =$$

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$$= \frac{10(4\sqrt{2} - 3\sqrt{3})}{16 \cdot 2 - 9 \cdot 3} - 3(3\sqrt{2} - 2\sqrt{3}) + 1 =$$

afăm semnul pt $2\sqrt{3} - 3\sqrt{2}$ prin compararea $2\sqrt{3}$ cu $3\sqrt{2}$

$$2\sqrt{3} = \sqrt{2^2 \cdot 3} = \sqrt{12} \quad ; \quad \text{cum } \sqrt{12} < \sqrt{18} \Rightarrow 2\sqrt{3} - 3\sqrt{2} < 0 \Rightarrow$$

$$3\sqrt{2} = \sqrt{3^2 \cdot 2} = \sqrt{18} \quad ; \quad \Rightarrow |2\sqrt{3} - 3\sqrt{2}| = -(2\sqrt{3} - 3\sqrt{2}) = 3\sqrt{2} - 2\sqrt{3}$$

$$= \frac{2 \cdot 10(4\sqrt{2} - 3\sqrt{3})}{5 \cdot 1} - 9\sqrt{2} + 6\sqrt{3} + 1 = 8\sqrt{2} - 6\sqrt{3} - 9\sqrt{2} + 6\sqrt{3} + 1 = \underline{\underline{1 - \sqrt{2}}}$$

$$(a-b)^{2021} = [-\sqrt{2} - (1 - \sqrt{2})]^{2021} = (-\sqrt{2} - 1 + \sqrt{2})^{2021} = (-1)^{2021} = -1, \text{ deoarece}$$

puterea 2021 este nr. impar.

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