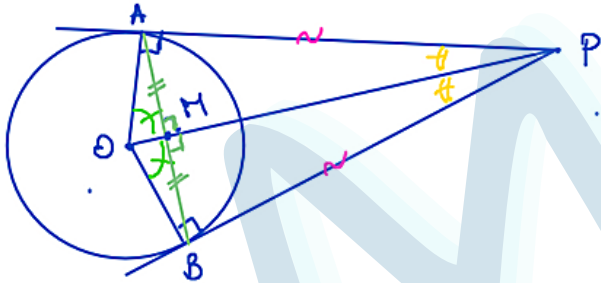


Mind Generation
 Centru de Matematica si Informatica

Matematica pentru clasa a VII-a
 Cercul - teorema ciocului de cioara

Teorema ciocului de cioara: Fie P un punct exterior unui cerc C (O, R) si punctele A si B punctele de contact ale tangentelor duse din P la cerc. Atunci:

- $[PA] \equiv [PB]$
- [PO este bisectoarea unghiului \widehat{APB}
- [OP este bisectoarea unghiului \widehat{AOB}
- OP este mediatoarea segmentului [AB]

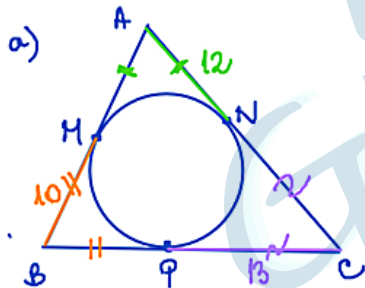


$C(O, R)$
 $P \in \text{Ext } C(O, R)$
 PA, PB tangente la $C(O, R)$

- $PA \equiv PB$
- $\widehat{APO} \equiv \widehat{BPO}$
- $\widehat{AOP} \equiv \widehat{BOP}$
- $OP \perp AB$
 Fie $AB \cap OP = \{M\} \Rightarrow$
 $\Rightarrow AM \equiv MB$

Probleme:

1. Aflati perimetrele triunghiurilor circumscrise cercurilor de mai jos:

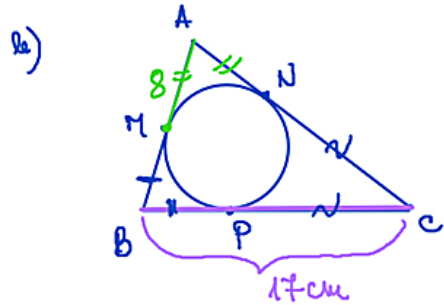


$MB = 10 \text{ cm} \Rightarrow BP = 10 \text{ cm}$
 $AN = 12 \text{ cm} \Rightarrow AM = 12 \text{ cm}$
 $PC = 13 \text{ cm} \Rightarrow CN = 13 \text{ cm}$

$$\Rightarrow P_{ABC} = AM + MB + BP + PC + AN + NC =$$

$$= 12 + 10 + 10 + 13 + 13 + 12 = 24 + 20 + 26 =$$

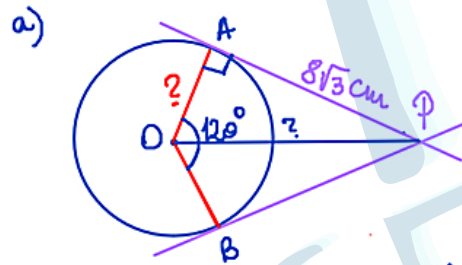
$$= \underline{\underline{70 \text{ cm}}}$$



$$\left. \begin{aligned} AM = 8 \text{ cm} &\Rightarrow AM = AN = 8 \text{ cm} \\ BC = 17 \text{ cm} &\Rightarrow BP + PC = 17 \text{ cm} \\ BP &\equiv BM \\ CP &\equiv CN \end{aligned} \right\} \Rightarrow BM + CN = 17 \text{ cm}$$

$$\Rightarrow P_{ABC} = \underbrace{AM}_{8} + \underbrace{BM}_{17} + \underbrace{BP + PC}_{17} + \underbrace{CN}_{8} + \underbrace{NA}_{8} = 2 \cdot 8 + 2 \cdot 17 = 2 \cdot 25 = \underline{50 \text{ cm}}$$

2. Prin punctul P exterior cercului de centru O se duc două tangente la cerc. Folositi datele din desen pentru a calcula lungimile necunoscute:



Jp: $\ell(O, r)$
 $\Rightarrow PA, PB$ tangente
 $PA = 8\sqrt{3} \text{ cm}; \hat{AOB} = 120^\circ$
 C: $OA = ?$
 $OP = ?$

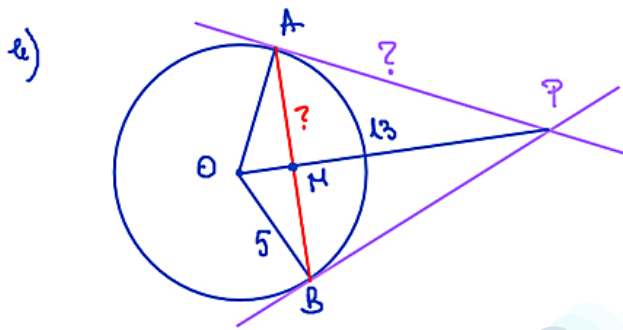
$$\left. \begin{aligned} A, B \text{ punctele de tangenta} &\Rightarrow \hat{OAP} = \hat{OBP} = 90^\circ \Rightarrow \\ \Rightarrow \Delta POA \left\{ \begin{aligned} \hat{OAP} &= 90^\circ \\ [OP \text{ bisectoare}] &\Rightarrow \hat{AOP} = \frac{120^\circ}{2} = 60^\circ \end{aligned} \right\} &\Rightarrow \hat{APO} = 30^\circ \xrightarrow{T. 30^\circ} OA = \frac{OP}{2} \\ AP &= 8\sqrt{3} \text{ cm} \end{aligned} \right\}$$

Notăm $OA = x \Rightarrow OP = 2x$

$$\text{Aplicăm T. P} \Rightarrow OP^2 = OA^2 + AP^2 \Rightarrow (2x)^2 = x^2 + (8\sqrt{3})^2 \Leftrightarrow$$

$$\Leftrightarrow 4x^2 = x^2 + 16 \cdot 3 \Leftrightarrow 3x^2 = 16 \cdot 3 \quad | :3 \Leftrightarrow x^2 = 16 \Rightarrow x = \sqrt{16} = \underline{4 \text{ cm}} \Rightarrow$$

Pag.2 $\Rightarrow OA = 4 \text{ cm}$ și $OP = 8 \text{ cm}$



Ip: PA, PB tangente $\mathcal{C}(O, R)$

$OB = 5 \text{ cm}$
 $OP = 13 \text{ cm}$

C: $AP = ?$
 $AB = ?$

D: PA, PB tangente $\Rightarrow PA = PB$ și $\hat{OAP} = \hat{OBP} = 90^\circ \Rightarrow \Delta OAP$ $\left\{ \begin{array}{l} \hat{OAP} = 90^\circ \\ OA = 5 \text{ cm} \\ OP = 13 \text{ cm} \end{array} \right.$ T.P. \Rightarrow

$OB = 5 \text{ cm} = R \Rightarrow OA = 5 \text{ cm}$

$\Rightarrow OP^2 = OA^2 + AP^2 \Rightarrow AP^2 = OP^2 - OA^2 = 13^2 - 5^2 = 169 - 25 = 144 \Rightarrow AP = \sqrt{144} = 12 \text{ cm}$

Fie $\{M\} = AB \cap OP \Rightarrow AM \perp OP$ T.Î. $\Rightarrow AM = \frac{OA \cdot AP}{OP}$ ($h_{\Delta \text{drept unghiic}} = \frac{\text{cateta 1} \cdot \text{cateta 2}}{\text{ipotenuză}}$)

$\Rightarrow AM = \frac{5 \cdot 12}{13} = \frac{60}{13} \text{ cm} \Rightarrow AB = 2 \cdot AM = 2 \cdot \frac{60}{13} = \frac{120}{13} \text{ cm}$