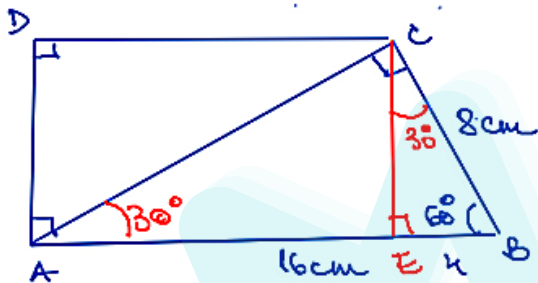


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~ Geometrie cls a VII-a ~
 Trapez - probleme rezolvate

1. Trapezul dreptunghic ABCD, $AB \parallel CD$, $AB > CD$, $m(\hat{A}) = 90^\circ$, are diagonala $AC \perp BC$, $m(\hat{ABC}) = 60^\circ$ si $BC = 8\text{cm}$. Calculati lungimile bazelor trapezului.



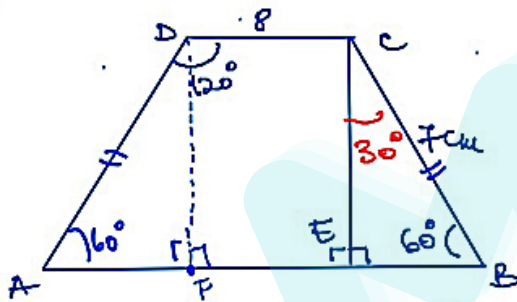
$\text{Ip: } ABCD \text{ trapez dr. } \Rightarrow C: AB = ?$
 $AB \parallel CD \Rightarrow DC = ?$
 $AB > CD$
 $m(\hat{A}) = 90^\circ$
 $AC \perp BC$
 $m(\hat{ABC}) = 60^\circ$
 $BC = 8\text{cm}$

Dem: $\Delta ABC \left\{ \begin{array}{l} \hat{ACB} = 90^\circ \\ \hat{ABC} = 60^\circ \end{array} \right. \Rightarrow \hat{CAB} = 30^\circ \left. \vphantom{\Delta ABC} \right\} \xrightarrow{T.30^\circ} CB = \frac{AB}{2}$
 $\Rightarrow AB = 2BC = 2 \cdot 8 = 16\text{cm}$
 Ducem $CE \perp AB$, $E \in (AB)$
 $\Delta CEB \left\{ \begin{array}{l} \hat{CEB} = 90^\circ \\ \hat{CBE} = 60^\circ \end{array} \right. \Rightarrow \hat{BCE} = 30^\circ \left. \vphantom{\Delta CEB} \right\} \xrightarrow{T.30^\circ} EB = \frac{CB}{2} \Rightarrow$
 $\Rightarrow EB = \frac{8}{2} = 4\text{cm} \Rightarrow AE = 16 - 4 = 12\text{cm} (*)$
 $\square AECD \left\{ \begin{array}{l} AD \perp AE, CE \perp AE \Rightarrow AD \parallel CE \\ DC \parallel AE \end{array} \right. \Rightarrow AECD \text{ dreptunghi} \Rightarrow$
 $\Rightarrow AE \equiv DC \left. \vphantom{\Rightarrow} \right\} \Rightarrow DC = 12\text{cm}$

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 Trapez - probleme rezolvate

2. In trapezul isoscel ABCD cu $AB \parallel CD$, $AB > CD$, se cunosc $CD=8\text{cm}$, $BC=7\text{cm}$ si $m(\widehat{ADC})=120^\circ$. Se construiesc $CE \perp AB$, $EE(AB)$.
 a) Calculati AE
 b) Calculati perimetrul trapezului



\hat{I}_p : ABCD
 Trapez isoscel
 $AB \parallel CD$
 $AB > CD$
 $m(\widehat{ADC}) = 120^\circ$
 $BC = 7\text{cm}$
 $CD = 8\text{cm}$

C:
 a) $AE = ?$
 b) $P_{ABCD} = ?$

Dem.: a) $m(\widehat{ADC}) = 120^\circ \Rightarrow m(\widehat{DCB}) = 120^\circ$ si $m(\widehat{A}) = 60^\circ$
 $\Rightarrow \widehat{ABC} = 60^\circ$
 $\Delta CEB \begin{cases} \widehat{CEB} = 90^\circ \\ \widehat{EBC} = 60^\circ \end{cases} \Rightarrow \widehat{ECB} = 30^\circ \xrightarrow{T.30^\circ} EB = \frac{BC}{2} \Rightarrow$
 $\Rightarrow EB = \frac{7}{2}\text{cm}$

Decem $DF \perp AB$, $FE(AB)$
 $\square DFEC \begin{cases} DF \perp AB, EC \perp AB \Rightarrow DF \parallel CE \\ DC \parallel FE \end{cases} \Rightarrow DFEC \text{ dreptunghi} \Rightarrow DC = FE \Rightarrow$
 $\Rightarrow FE = 8\text{cm}$

$\Delta ADF \cong \Delta BCE \begin{cases} AD = CB (\hat{I}_p) \\ \widehat{DFA} = \widehat{CEB} = 90^\circ \\ \widehat{DAF} = \widehat{CBE} = 60^\circ \end{cases} \Rightarrow AF = EB$
 $(i.u.) \quad EB = \frac{7}{2}\text{cm} \Rightarrow AF = \frac{7}{2}\text{cm} \Rightarrow$

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$\Rightarrow AE = AF + FE = \frac{7}{2} + 8 = \frac{7+16}{2} = \frac{23}{2}\text{cm}$

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Trapez - probleme rezolvate

$$b) P_{ABCD} = AB + BC + CD + DA$$

$$AB = AF + FE + EB$$

$$AF = EB = \frac{7}{2} \text{ cm}$$

$$FE = 8 \text{ cm}$$

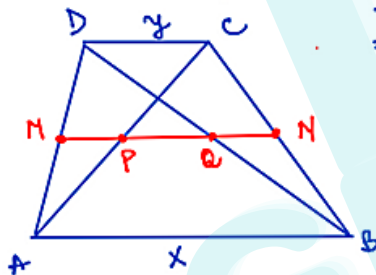
$$\Rightarrow AB = \frac{7}{2} + \frac{7}{2} + 8 = 15 \text{ cm}$$

$$BC = DA = 7 \text{ cm (trapez isoscel)}$$

$$DC = 8 \text{ cm}$$

$$\Rightarrow P_{ABCD} = 15 + 2 \cdot 7 + 8 = \underline{37 \text{ cm}}$$

3. Intr-un trapez, lungimea liniei mijlocii este de 5 cm, iar lungimea segmentului determinat de mijloacele diagonalelor este de 2 cm. Determinati lungimile bazelor trapezului.



Jp: $\square ABCD$ trapez
 $M \in (AD), AM \equiv MD$
 $N \in (BC), NB \equiv NC$
 $P \in (AC), PA \equiv PC$
 $Q \in (BD), QB \equiv QD$
 $MN = 5 \text{ cm}$
 $PQ = 2 \text{ cm}$

C : $DC = ?$
 $AB = ?$

Demi: Notam $AB = x, DC = y$; $[MN]$ = linie mijl $\Rightarrow MN = \frac{AB + DC}{2}$ (1)
 $\Rightarrow \frac{x + y}{2} = 5 \Leftrightarrow x + y = 10$ (*)
 $\S MN \parallel DC$ (2)
 ΔADC { M mijloc $[AD]$ $\Rightarrow [MP]$ = lin. mijlocie $\Rightarrow MP = \frac{DC}{2} = \frac{y}{2}$ (3)
 P mijloc $[AC]$ $\S MP \parallel DC$ (4)
 ΔBDC { N mijloc $[BC]$ $\Rightarrow [QN]$ = lin. mijlocie $\Rightarrow QN = \frac{DC}{2} = \frac{y}{2}$ (5)
 Q mijloc $[BD]$ $\S QN \parallel DC$ (6)
 Din (2), (4), (6) $\Rightarrow M, P, Q, N$ coliniare
 Din (3), (5) $\Rightarrow PQ = MN - MP - QN = \frac{x + y}{2} - \frac{y}{2} - \frac{y}{2} = \frac{x - y}{2} \Rightarrow$
 $\Rightarrow \frac{x - y}{2} = 2 \Leftrightarrow x - y = 4$ (**)

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Trapez - probleme rezolvate

$$\Rightarrow \text{din } (*) \text{ si } (**): \begin{array}{l} x+y=10 \\ x-y=4 \end{array} \quad \textcircled{+} \text{ Adunăm relațiile}$$
$$\frac{2x}{2} = 14 \Rightarrow x = \frac{14}{2} = 7 \text{ cm} \Rightarrow \underline{AB = 7 \text{ cm}}$$
$$y = 10 - x = 10 - 7 = 3 \text{ cm} \Rightarrow \underline{DC = 3 \text{ cm}}$$

Probă: $MN = \frac{7+3}{2} = 5 \text{ cm} \checkmark$

$PQ = \frac{7-3}{2} = 2 \text{ cm} \checkmark$

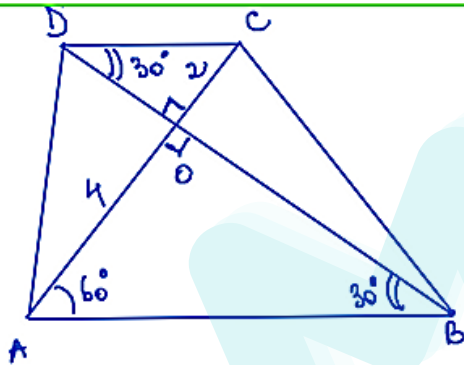
Retinem: Segmentul care uneste mijloacele diagonalelor unui trapez este semidiferența dintre baza mare și baza mică!

$$PQ = \frac{AB - CD}{2}$$

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 Trapez - probleme rezolvate

4. In trapezul ABCD ($AB \parallel CD$, $AB > CD$) diagonalele AC si BD sunt perpendiculare. Stiind ca $m(\widehat{CAB}) = 60^\circ$, $AC = 6\text{cm}$ si $AO = 2 \cdot OC$, unde O este punctul de intersectie al diagonalelor, calculati lungimile bazelor trapezului.



Jp: ABCD trapez
 $\Rightarrow AB \parallel CD, AB > CD$
 $AC \perp BD$
 $\widehat{CAB} = 60^\circ$
 $AC = 6\text{cm}$
 $AO = 2 \cdot OC$
 $\{O\} = AC \cap BD$
 C: $DC = ?$
 $= AB = ?$

Dem: $AC = 6\text{cm}$
 $AO = 2 \cdot OC \Rightarrow AC = 2OC + OC = 3OC \Rightarrow 3OC = 6 \Rightarrow OC = 2\text{cm}$
 $\& AO = 4\text{cm}$
 $\Delta AOB \left\{ \begin{array}{l} \widehat{AOB} = 90^\circ \\ \widehat{OAB} = 60^\circ \end{array} \right. \Rightarrow \left. \begin{array}{l} \widehat{OBA} = 30^\circ \\ AO = 4\text{cm} \end{array} \right\} \xrightarrow{T.30^\circ} AB = 2AO \Rightarrow AB = 8\text{cm}$
 $AB \parallel DC$
 $DB \text{ secanta } \left. \begin{array}{l} \Rightarrow \widehat{CDB} \equiv \widehat{ABD} \text{ (alt. int. congr.)} \\ \Rightarrow \widehat{CDB} = 30^\circ \end{array} \right\} \Rightarrow$
 $\Rightarrow \Delta DOC \left\{ \begin{array}{l} \widehat{DOC} = 90^\circ \\ \widehat{DCO} = 30^\circ \\ OC = 2\text{cm} \end{array} \right\} \xrightarrow{T.30^\circ} \left. \begin{array}{l} DC = 2 \cdot OC \\ OC = 2\text{cm} \end{array} \right\} \Rightarrow DC = 4\text{cm}$